

The M-theory Path Integral and its Ensembles

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Reykjavik, June 2026

Based on

[Phys. Rev. Lett. 135 \(2025\) 10, 101601](#) - Fridrik Gautason, JvM

[JHEP 11 \(2025\) 078](#) - Fridrik Gautason, JvM

[\[2603.14544\]](#) - JvM

and ongoing work with Nikolay Bobev, Fridrik Gautason, Jieming Lin, Zihan Wang, and Sam Bennett

Goal: M-theory

Method: Holography as a benchmark

Outline:

Gravity



Strings



Membranes



Ensembles

A useful probe into quantum gravity is the Euclidean gravitational path integral

$$Z_{\text{QG}} = \sum_{\mathcal{M}} \int \mathcal{D}[g] e^{-I[g]}, \quad I[g] = \frac{1}{16\pi G_N} \int d^d x \sqrt{g} (-R + 2\Lambda)$$

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A **microscopic** approach resolves the issues \rightarrow **holography**

$$\boxed{\mathcal{Z}_{\text{QG}} = Z_{\text{QFT}}}$$

Holography can be studied quantitatively in string theory

$$\boxed{\mathcal{Z}_{\text{string}} = \log Z_{\text{QFT}}}$$

where we have microscopic control on the LHS

$$\mathcal{Z}_{\text{string}}[g, B_2, \dots] = \sum_{\chi} \frac{1}{g_s^\chi} \int \mathcal{D}[h, \phi, \psi] e^{-h^{ab} g_{\mu\nu} \partial_a \phi^\mu \partial_b \phi^\nu + \dots},$$

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Two distinct non-perturbative sectors to be included in the string path integral, and in holography

$$\mathcal{Z}_{\text{string}} = -S_{10d} + \dots + \sum_{\text{WS}} e^{-S_{\text{WS}}(\dots)} + \sum_{\text{D}} e^{-S_{\text{D}}(\dots)} = \log Z_{\text{QFT}}$$

Gautason, Puletti, JvM '23; Gautason, JvM '24

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This holographic equality now needs to be taken with care, with a correct map between background sources, and leads to different holographic ensembles on the boundary!

Gautason, JvM '25

An M-theory Example

Ensembles in AdS_4

$$\text{AdS}_4 \times S^7 / \mathbf{Z}_k \quad \longleftrightarrow \quad \text{ABJM theory with } \text{U}(N)_k \times \text{U}(N)_{-k}$$

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Bulk: two consistent boundary conditions

$$\int \star G_4 \quad \text{or} \quad \int A_3$$

Boundary: fix gauge rank, or canonically conjugate potential

$$\exp[N] \quad \text{or} \quad \exp[\mu]$$

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Careful:

- Bulk is naturally studied at fixed gauge potential

QFT in grand canonical ensemble

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Holographic comparisons require a change in ensembles!

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$$Z[\star G_4] = \int \mathcal{D}A_3 e^{\mathcal{Z}_{\text{M2}}[A_3]} e^{\int A_3 \wedge \star G_4}$$

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In terms of the quantised flux and chemical potential

$$Z_{\text{QFT}}[N] = \oint d\mu Z_{\text{M}}[\mu] e^{\mu N}, \quad \text{where} \quad Z_{\text{M}}[\mu] = \sum_n e^{\mathcal{Z}_{M2}[\mu + 2\pi i n]}$$

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What remains are **quantitative checks through holography**

ABJM on: S^3 , S_b^3 , $S^2 \times S^1$, $\Sigma_g \times S^1$

SUGRA: 2∂ , 8∂ , **logs**, membrane instantons and defects

with Bobev, Gautason

Example: the boundary S^3 partition function

Drukker, Fuji, Hirano, Hatsuda, Marino, Putrov, Moriyama, Okuyama, ...

$$F_{\text{QFT}}^{\text{CE}}[N] = \frac{-2N^{3/2}}{3C_k^{1/2}} + \frac{B_k N^{1/2}}{C_k^{1/2}} - \frac{\log N}{4} - \frac{\log\left(\frac{16\pi^2 C_k}{e^{4A_k}}\right)}{4} + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$

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Two Derivative Supergravity:

$$S_{2\partial}(A_3) = \frac{2}{3\pi^2 k} \frac{L^9}{\ell_p^9} = \frac{C_k}{3} \mu^3 \neq \frac{-2N^{3/2}}{3C_k^{1/2}}$$

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The *canonical* answer is found through the Laplace transform which can be expanded at large μ

$$S_{2\partial}(\star G_4) = S_{2\partial}(A_3) - \int_{\partial} A_3 \wedge \star G_4 = -\frac{1}{3\pi^2 k} \frac{L^9}{\ell_p^9} = \frac{-2N^{3/2}}{3C_k^{1/2}}$$

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Eight Derivative Supergravity:

$$S_{8\partial}(A_3) = \int A_3 \wedge X_8 + \dots \quad \leftarrow \quad \textit{unknown}$$

Instead a 4d gauged supergravity approach can be taken.

The canonical answer is found by expanding the Laplace transform to higher order in μ .

No further higher derivative corrections for μ !

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Logs:

1. Loop divergences (absent in 11d)
2. Zero-modes

The counting of zero-modes is **IR universal** and can be derived from the two-derivative supergravity action.

Bhattacharyya, Grassi, Marino, Sen '12

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The *normalizable* zero-mode comes from a **two-form ghost** in A_3 , which for us has Dirichlet boundary conditions!

$$\delta A_3|_{\partial} = 0 \quad \Rightarrow \quad \lambda_2|_{\partial} = 0, \quad \text{for} \quad A_3 \rightarrow A_3 + d\lambda_2$$

Associated ghosts vanish on the boundary $\rightarrow 0 \log \mu!$

Summary

$$Z_{\text{QFT}}[N] = \oint d\mu Z_{\text{M}}[\mu] e^{\mu N}$$

- ▶ Bulk partition functions need boundary conditions
- ▶ The boundary theory is in an ensemble

Extra

- ▶ Instantons and defects are 1-loop exact

Future

- ▶ What about M5-branes in AdS₄? with Z. Wang & J. Lin
- ▶ Generalizations: $S^7 \rightarrow SE_7$ with S. Bennett
- ▶ Ensembles in $\mathcal{N} = 4$ SYM, 6d (2, 0) theory, ...