Matrix Theory, String Worldsheet, and Supergravity

Ziqi Yan

Center of Gravity, Niels Bohr Institute

[Maskalaniec, ZY, Zorba '26]

[de Boer, Dijkgraaf, Harmark, Obers, ZY, to appear]

[Baiguera, Harmark, Lei, Makalaniec, ZY, to appear]

34th Nordic Network Meeting

University of Iceland

June 2nd, 2026

Olle Engkvist Foundation

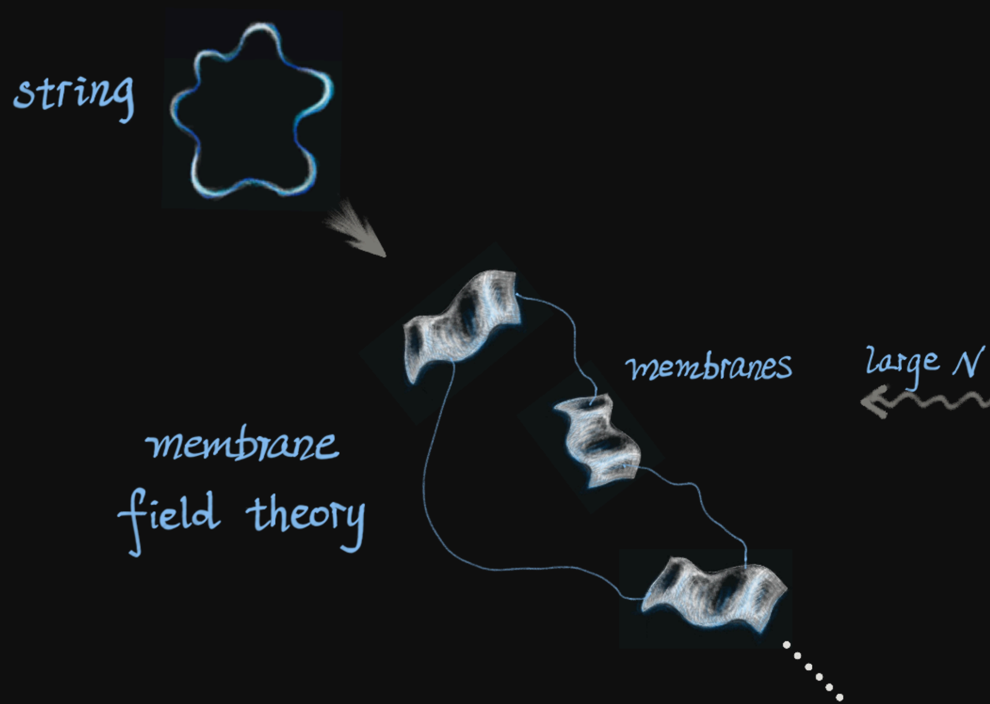
Funded by Villum Young Investigator Program & Olle Engkvists Stiftelse

VILLUM FONDEN

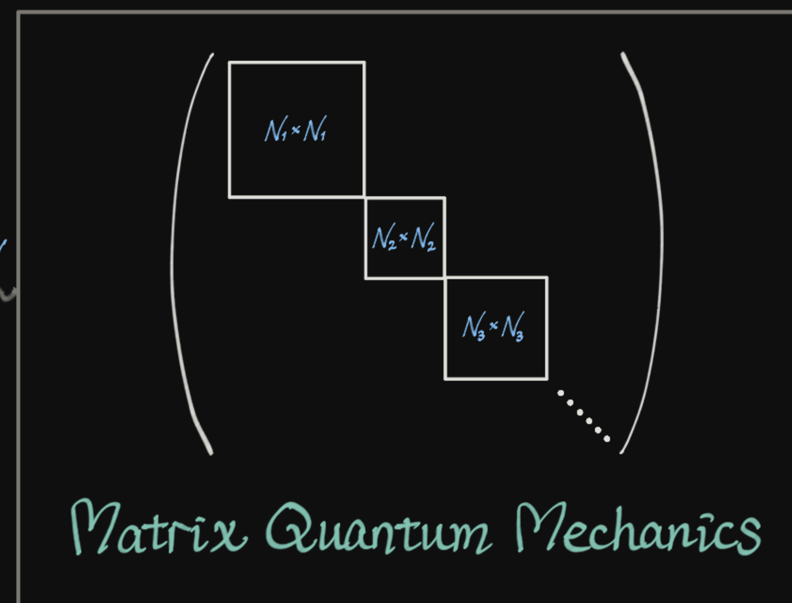
How (quantum) gravity emerges from quantum mechanical systems?

string theory: 2D quantum sigma models = 10D quantum gravity

non-perturbative regime: M-theory ?



[de Wit, Hoppe, Nicolai '88]
[Banks, Fischler, Shenker, Susskind '96]



10D perspective: D-brane formalism for target space physics ?

BFSS matrix theory from a non-relativistic limit

$N=1$: a single D-particle in type IIA superstring theory

i.e. relativistic particle coupled to $U(1)$ gauge

$$S = -mc\int d\tau \sqrt{c^2\dot{X}_0^2 - \dot{X}_i^2} + \frac{1}{\ell_s}\int \left(C^{(1)} = c^2 g_s^{-1} dX^0\right)$$

$$= \int d\tau \left(-mc^2 + \frac{1}{2}m\dot{X}_i^2 + mc^2\right) + O(c^{-2})$$

$$\xrightarrow{c\to\infty} \frac{1}{2}m\int d\tau\, \dot{X}_i^2$$

decoupling limit zooming in on a BPS state

decoupling limit $c\to\infty$

$$ds_{10}^2 \to -c\, dX_0^2 + \frac{1}{c}dX_i^2$$

$$g_s \to c^{-\frac{3}{2}} g_s$$

$$C^{(1)} \to c^2 g_s^{-1} dX^0$$

N coinciding D-particles: $\mathcal{N}=16$ supersymmetric quantum mechanics of 9 $N\times N$ matrices

$$S_{BFSS} = \frac{1}{g_s l_s}\int d\tau\, \mathrm{tr}\left[\frac{1}{2}D_\tau X^i D_\tau X^i + \frac{1}{4}\ell_s^{-4}[X^i, X^j]^2 + \frac{1}{2}\Psi^T\left(iD_\tau\Psi + \ell_s^{-2}\gamma_i[X^i,\Psi]\right)\right]$$

effective coupling of BFSS

$$g_{\text{eff}}^2 \sim \frac{N\, g_{YM}^2}{E^3} \qquad g_{YM}^2 = \frac{g_s}{\ell_s^3}$$

ground state open string

$$E \sim \frac{r}{\ell_s^2}$$

instantaneous Newton-like force

$$g_{\text{eff}}^2 \sim N \ell_s^3 \frac{g_s}{r^3} \longrightarrow N \ell_s^3 \frac{c^{-\frac{3}{2}} g_s}{(r/\sqrt{c})^3} \longrightarrow \text{fixed}$$

decoupling limit $c \to \infty$

$$ds_{10}^2 \to -c\, dt^2 + \frac{1}{c}\left(dr^2 + r^2 d\Omega_8^2\right)$$

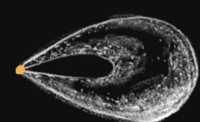
$$g_s \to c^{-\frac{3}{2}} g_s$$

$$C^{(1)} \to c^2 g_s^{-1}\, dt$$

$$\frac{r}{\ell_s} \to \frac{r/\sqrt{c}}{\ell_s} \longrightarrow 0$$

substring regime [Shenker '95]

"fundamental" string: non-vibrating



$$S = \frac{T}{2} \int d^2\sigma \left(\lambda\, \partial_\tau X^0 + \partial_\sigma X^0 \partial_\sigma X^0 + \partial_\tau X^i \partial_\tau X^i \right)$$ [Gomis, ZY '23]

D2-branes: 3D non-commutative Yang-Mills theory

target space: geometric optics [Batlle, Gomis, Not '16] [Gomis, Townsend '16] [Batlle, Gomis, Mezincescu, Townsend '17]

topological twist → tropical geometry & Gromov-Witten [Albrychiewicz, Ellers, Hořava '23] decoupling limit of type IIA superstring theory:

dilatation symmetry

$$\tau \to \Delta^{\frac{1}{2}} \tau \qquad E^i \to \Delta^{-\frac{1}{2}} E^i$$

$$e^{\varphi} \to \Delta^{-\frac{3}{2}} e^{\varphi}$$

Matrix 0-brane Theory (M0T)

flat background

$$ds_{10}^2 \to -\omega\, dt^2 + \frac{1}{\omega}\left(dr^2 + r^2 d\Omega_8^2\right)$$

$$g_s \to \omega^{-\frac{3}{2}} g_s \qquad \omega \to \infty$$

11D Lorentz factor for infinite boost

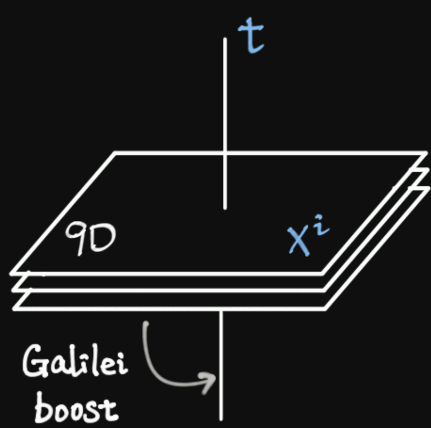
$$C^{(1)} \to \omega^2 g_s^{-1}\, dt$$

curved background

$$G_{\mu\nu} = -\omega\, \tau_\mu \tau_\nu + \frac{1}{\omega} E_\mu{}^i E_\nu{}^i \qquad i = 1, \cdots, 9$$

$$e^{\Phi} = \omega^{-\frac{3}{2}} e^{\varphi} \qquad \omega \to \infty$$

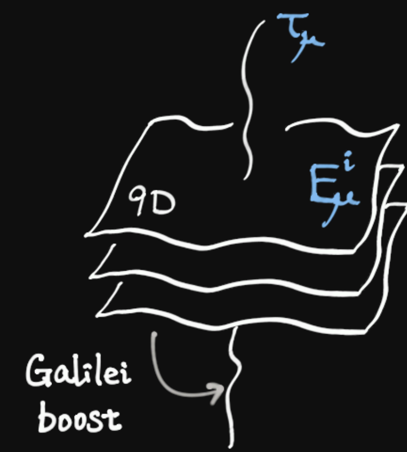
$$C^{(1)} = \omega^2 e^{-\varphi} \tau_\mu dx^\mu + c^{(1)}$$



$$\delta t = 0$$

$$\delta x^i = v^i t$$

Newton-Cartan formalism



$$\delta \tau_\mu = 0$$

$$\delta E_\mu{}^i = v^i \tau_\mu$$

[Blair, Lahnsteiner, Obers, ZY ’23 ’24]

[Maskalaniec, ZY, Zorba '26]

BFSS at low N: D-particle backreaction can be ignored

a non-rel. quantum gravity

$$S_{IIA} = \frac{1}{2\kappa^2}\int d^{10}x \sqrt{-G}\left[e^{-2\Phi}\left(R + 4\partial_\mu\Phi\partial^\mu\Phi\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

$\omega \to \infty$ $\qquad F^{(2)} = dC^{(1)}$

[Hartong, Obers, Oling '22]
[Lambert, Smith '24]
[Bergshoeff, Giorgio, Romano '24]

$$S_{NR} = \frac{1}{2\kappa^2}\int d^{10}x \det(\tau_\mu\, E_\mu{}^i)\, e^{-2\varphi}\, E^{\mu\nu}\Big(r_{\mu\nu} - 4\tau^\rho\nabla_\mu\partial_{[\nu}\tau_{\rho]} - 6\tau^\rho\tau^\sigma d_{[\mu}\tau_{\rho]}d_{[\nu}\tau_{\sigma]} - e^{\varphi}E^{\rho\sigma}d_{[\mu}\tau_{\rho]}d_{[\nu}C_{\sigma]}\Big)$$

null reduction of 11D supergravity

dilatation covariant derivative

$$d_\mu \mathcal{O} = \left(\partial_\mu + \frac{2}{3}\Delta_{\mathcal{O}}\,\partial_\mu\varphi\right)\mathcal{O}$$

compatibility conditions

$$\nabla_\mu\tau_\nu = d_\mu\tau_\nu - \gamma^\rho{}_{\mu\nu}\tau_\rho = 0$$

$$\nabla_\mu E^\nu{}_i = d_\mu E^\nu{}_i + \gamma^\nu{}_{\mu\rho}E^\rho{}_i = 0$$

Newton-Cartan connection $\qquad E_{\mu\nu} = E_\mu{}^i E_\nu{}^i$

$$\gamma^\mu{}_{\rho\sigma} = \tau^\mu d_\rho\tau_\sigma + \frac{1}{2}E^{\mu\nu}\left(d_\rho E_{\sigma\nu} + d_\sigma E_{\rho\nu} - d_\nu E_{\rho\sigma}\right)$$

Newton-Cartan Riemann tensor

$$r^\rho{}_{\sigma\mu\nu} = d_\mu\gamma^\rho{}_{\nu\sigma} - d_\nu\gamma^\rho{}_{\mu\sigma} + \gamma^\rho{}_{\mu\lambda}\gamma^\lambda{}_{\nu\sigma} - \gamma^\rho{}_{\nu\lambda}\gamma^\lambda{}_{\mu\sigma}$$

dynamics?

an example... backreact D4-branes: non-Lorentzian geometry in MOJ

$$\tau_\mu dx^\mu = \frac{dt}{H_4^{1/4}} \qquad E_{\mu\nu} dx^\mu dx^\nu = \frac{dx_1^2 + \cdots + dx_4^2}{H_4^{1/2}} + H_4^{1/2}\left(dr^2 + r^2 d\Omega_4^2\right)$$

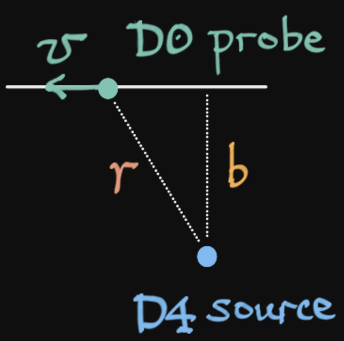
$$e^{\varphi} = \frac{g_s}{H_4^{1/4}} \qquad C^{(5)} = \frac{1}{g_s H_4}\, dt \wedge \cdots \wedge dx^4 \qquad H_4 = 1 + \frac{\ell^3}{r^3}$$

a probe D-particle

$$S = \frac{1}{2\ell_s}\int d\tau\, e^{-\varphi}\, \frac{\dot{x}^\mu \dot{x}^\nu E_{\mu\nu}}{\dot{x}^\rho \tau_\rho} + \int C^{(1)} = \frac{1}{2 g_s \ell_s}\int d\tau \left(1 + \frac{\ell^3}{r^3}\right) v^2$$

eikonal scattering between D0 & D4 in IIA theory

effective potential $\sim \frac{v^2}{r^3}$



$$\delta(b, v) = \int_0^\infty \frac{ds}{s}\, e^{-s b^2} \tan\frac{s v}{2}$$ [Lifschytz '96]

$$\Rightarrow\ S = \frac{1}{2 g_s \ell_s}\int d\tau \left(1 + \frac{\ell^3}{r^3}\right) v^2 + \mathcal{O}\left(\frac{v^4}{\ell^7}\right)$$

[Danielsson, Ferretti, Sundborg '96] [Kabat, Pouliot '96] [Douglas, Kabat, Pouliot, Shenker '96]

D-brane generalization of $T\bar{T}$

backreact D-particles: deform MOT to IIA

$$\Box w^{-2} \sim \delta^{(9)}(x^i) \quad \Rightarrow \quad w = \sqrt{\frac{r^7}{\ell^7}} \qquad \ell^7 = N g_s \ell_s^7$$

$$ds_{10}^2 = -w\, dt^2 + \frac{1}{w} dx_i^2$$

$$e^{\Phi} = w^{-\frac{3}{2}} g_s$$

$$C^{(1)} = w^2 g_s^{-1} dt$$

deform away from non-Lorentzian corner

bulk geometry with conformally AdS_2 & S^8

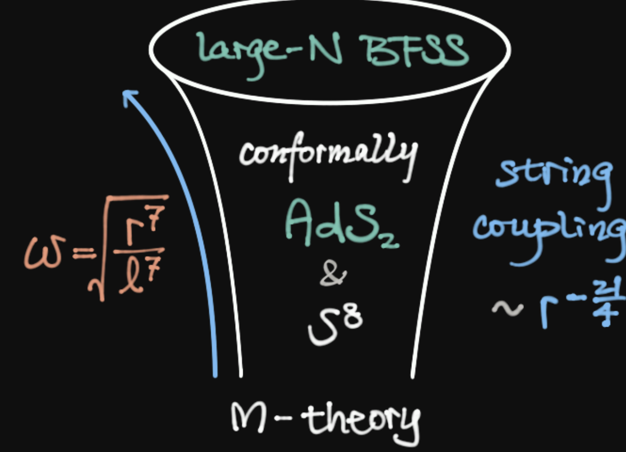
classical supergravity dominates when

$$N^{\frac{1}{7}} \ll \frac{r}{\ell_{11}} \ll N^{\frac{1}{3}} \qquad \ell_{11} = g_s^{\frac{1}{3}} \ell_s$$

$$ds_{10}^2 = -\left(\frac{r}{\ell}\right)^{\frac{7}{2}} dt^2 + \left(\frac{\ell}{r}\right)^{\frac{7}{2}} (dx_1^2 + \cdots + dx_9^2)$$

$$e^{\Phi} = \left(\frac{\ell}{r}\right)^{\frac{21}{4}} g_s$$

$$C^{(1)} = \left(\frac{r}{\ell}\right)^{14} g_s^{-1} dt$$



[Itzhaki, Maldacena, Sonnenschein, Yankielowicz ’98]

IIA string theory in decoupling limit

$g_s \lesssim 1$, low N, no D-particle backreaction: quantum mechanics dominates

MOT (w/ perturbative BFSS) $\sim$ 10D non-Lorentzian quantum gravity

large g_s, large N, no D-particle backreaction

strongly coupled BFSS $\sim$ M-theory with null isometry

small g_s, large N, D-particles backreact $N^{\frac{1}{7}} \ll \frac{r}{\ell_{11}} \ll N^{\frac{1}{3}}$, $\ell_{11} = g_s^{\frac{1}{3}} \ell_s$

classical sugra on conformally AdS_2 dominates

holographic correspondence: string theory in decoupling limit is dominated by

quantum mechanical dynamics in one regime ($g_s \lesssim 1$, low N) $\rightsquigarrow$ MOT

classical Lorentzian gravity in another regime (small g_s, large N)

in what sense gravity is emergent? non-vibrating string in MOT; β-functions?

[Maskalaniec, ZY, Zorba '26]

analog of β-functions?

back to F-string: $S = \int d^2\sigma \left(\lambda \partial_\tau X^0 + \partial_\sigma X^0 \partial_\sigma X^0 + \partial_\tau X^i \partial_\tau X^i\right)$ string tension $T=2$

1st-order formulation: $S = \int d^2\sigma \left[P_\mu \partial_\tau X^\mu - \frac{\chi}{4}\left(P_i P_i - 4 \partial_\sigma X^0 \partial_\sigma X^0\right) - \rho P_\mu \partial_\sigma X^\mu \right]$

ambitwistor string gauge: $\chi = 0,\ \rho = 1 \Rightarrow S = \int d^2\sigma\, P_\mu \bar\partial X^\mu$ $\quad \bar\partial = \partial_\tau - \partial_\sigma$

Hamiltonian constraint $\mathcal{H} = P_i P_i - \partial X^0 \partial X^0 \sim 0$

chiral superstring $S = \int d^2\sigma \left(P_\mu \bar\partial X^\mu + \bar\psi_\mu \bar\partial \psi^\mu\right)$

supercurrents $G = \psi^i P_i + \psi^0 \bar\partial X^0 \sim 0$

$\bar G = \bar\psi_i P_i - \bar\psi_0 \partial X^0 \sim 0$

$$G(z)\,\bar G(w) \sim \frac{\mathcal{H}(z)}{z-w}$$

[Adamo, Casali, Skinner '14]

arbitrary background fields: curved $\beta\gamma$ system $\longrightarrow$ anomalies in current algebra

anomaly-free conditions: target space e.o.m.?

Thank You!